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Frege's logicism

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FREGE'S LOGICISM

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FREGE'S LOGICISM

JARED HENDERSON

ABSTRACT

In this paper, I provide an interpretation of Frege's logicist project, drawing a connection between it and his idiosyncratic view of truth.

Contents

Chapter 1

Frege's Logicism

1.1 Introduction

Gottlob Frege is perhaps the most important philosopher of late 19th and early 20th century. With the introduction of his *begriffsschrift*, Frege realized a goal imagined at least as early as Leibniz, that of a universal language, free of ambiguity, that could be utilized by logicians, mathematicians, and scientists in their endeavors—thus allowing for progress unhindered by talking past one another or failure to understand.[1] If there is a failure to understand, it is caused in one of two ways: either the writer using the *begriffsschrift* has used it improperly, or the reader fails to fully understand the *begriffsschrift*.

Though his notation is no longer in widespread use, Frege's innovation has allowed for the development of an incredibly fruitful area of research in the 20th and 21st centuries: formal semantics. Frege also broke new ground with his thesis of compositionality, which holds that the meaning of a sentence is determined by the words in the sentence and their ordering. These two insights allowed for a scientific study of language.

These insights, however, were all in service of a larger project, one which had little to with language in and of itself. In what follows, I give a characterization of Frege's project, the project which he worked on from at least the publication of the *Begriffsschrift* in 1879 until his death in 1925. I intend to show what Frege was, so to speak, up to, for almost half a century. This is not to imply that there was little or no change in Frege's

[1] '*Begriffsschrift*' ought to be capitalized. However, following Richard Heck [12], I will write '*begriffsschrift*' when referring to Frege's formal language, and '*Begriffsschrift*' when referring to Frege's publication. I pay the price of clarity with the coin of German grammar.

particular views. Frege held, e.g., that signs 'refer to themselves' in the context of an identity statement when writing *Begriffsschrift*, but held a more orthodox view of identity (in which the signs refer to their designata) by the time "On *Sinn* and *Bedeutung*" was published in 1892.[2] Rather, what I intend to show is that Frege was a logicist, and in fact a particular kind of logicist.

1.1.1 Two types of logicism

The first claim is obvious. If 'logicism' is understood simply as the view that arithmetic is an extension of logic, or that "the theory of number is a part of logic" [9], then there is little need to argue that Frege is a logicist—he is the paradigmatic logicist. *Basic Laws of Arithmetic* attempts to show that the laws of arithmetic may be derived from a minimal number of laws of logic by the means of proof.[3] Enough said, then: Frege is a logicist. However, in the literature, it is recognized that there are a variety of logicisms, and so when characterizing Frege's project, we must be more detailed.[4]

A natural way to understand the taxonomy of logicisms is to partition the views into two broad camps: metaphysical and epistemological. The distinction comes, at least partly, from Shapiro [20]. Shapiro holds that there are at least three ways in which one can provide a foundation for mathematics: metaphysical, epistemological, and structural.[5] However, Shapiro claims that that the metaphysical kind of foundations is ontological—that is, it provides the required ontology for various branches of mathematics. This is one way to provide a foundation, though my characterization of metaphysical logicism is about priority, not merely ontology.[6]

[2] Frege allows himself to have an orthodox view of identity statements by adopting the *Sinn*/*Bedeutung* distinction—his previous view of identity sought to retain a one-dimensional view of semantic value while also honoring the informativity of identity statements.

[3] And in fact, Frege helped himself only to a minimal proof system, featuring only universal quantification, negation, the material conditional, and the inference rule *modus ponens.*

[4] I have in mind here the distinction between 'weak' and 'strong' logicism. The strong logicist holds that all mathematical truths are derivable from logic alone; the weak logicist restricts the claim to theorems. C.f. Tennant [22].

[5] I will be ignoring the structural approach throughout.

[6] For more on priority claims, c.f. Schaffer [19].

Metaphysical logicism holds something like the following view:

> GROUND: The laws of arithmetic are grounded in, or made true by, the laws of logic. By giving proofs from the laws of logic to the laws of arithmetic, we show that the laws of logic are the truth-makers for the laws of arithmetic.

A metaphysical logicist will begin his project by asking: In virtue of what do mathematical or arithmetical truths hold? The question is then about truths and their truth-makers.[7] The answer: mathematical truths are grounded in the laws of logic.[8] This leads the logicist to then endorse GROUND. Though perhaps anachronistic in its terminology, GROUND is a plausible thesis on its own and, as I briefly show in the following section, a plausible line of interpretation of Frege's corpus.[9] In contrast, epistemological logicism holds something like the following:

> JUSTIFICATION: Belief in the laws of arithmetic is justified insofar as we can show that they follow from the laws of logic. By giving proofs from justified logical premises to general arithmetical conclusions, we become justified in fully believing the laws of arithmetic.

An epistemological logicist will begin his project by asking something very different from the metaphysical logicist. He asks: in virtue of what am I justified in believing that the laws of arithmetic hold? And if the epistemological logicist is anything like Frege (which he ought to be, since I hold that Frege is an epistemological logicist), then he'll want as

[7] Or, perhaps, about grounding and reduction. Horsten [14], e.g., sees Frege as providing a *reduction* of arithmetic to logic. However, this use of 'reduction' can be misleading. Reductions can either explain priority or eliminate the reduced from our ontology. It is not clear that either of these is what a logicist ought to see himself as doing. See Rosen [18] and Schaffer [19] for a general introduction to grounding and reduction.

[8] It isn't immediately obvious what kinds of things ground what kinds of things—that is, if grounding is a relation between objects, between facts or states of affairs, or propositions. I hedge on this issue, though I'll use the parlance of truths (understood as true propositions) being grounded in other true propositions.

[9] A quick note about the alleged anachronism: if some metaphysicians are to be believed, grounding-talk has been around throughout philosophical history, even if the particular term 'grounding' was not used. Additionally, it seems plausible that some of Frege's contemporaries and immediate forebears were thinking about grounding (e.g. Bolzano, who spoke of 'objective ground' in logic), See Correia and Schneider [2] for a historical and theoretical overview of grounding.

much justification as possible, because he thinks that arithmetic must be beyond doubt. JUSTIFICATION is then the natural view.

1.1.2 Mapping the argument

I will argue throughout that Frege is an epistemological logicist. However, in order to argue this point, I will have to argue for several other points along the way. This requires that I make clear what I think Frege believes, particularly, about the nature of truth and the character of the laws of logic. I will also keep in mind Frege's critiques of psychologism, arguing in part that his method of attack against the psychologistic logicians lends further support for the interpretation I am putting forth.

Before turning to the interpretation, I want to make several notes about the taxonomy provided above. First, I do not take GROUND and JUSTIFICATION to be exclusive, though they are independent. To my mind, it seems natural for a metaphysical logicist who endorses GROUND to also endorse JUSTIFICATION, and thus also be an epistemological logicist. Carving up the views in this way, however, leaves open the possibility that one can be concerned only with truth-making or only with justification. Since the two are distinct, this is a virtue. Further, since Frege is—as I argue in §1.2— a truth-making skeptic, we'll want to keep truth-making and justification separate in any interpretation of Frege. Finally, I take my metaphysical/epistemological distinction to be independent from the weak/strong logicism distinction. Thus, one can be a weak or strong metaphysical logicist and a weak or strong epistemological logicist. I think Frege would particularly be a strong epistemological logicist, but I do not argue for this particular point.

I'll now sketch the argument. In §1.2, I clear up an initial confusion, namely that because Frege has metaphysical commitments (particularly, his position that numbers are objects) he is therefore a metaphysical logicist. To think that would be, again, a confusion, since I stipulated that 'metaphysical logicism' here is just a commitment to GROUND. But thinking that Frege would endorse GROUND because of his ontology of numbers would actually lead to a rejection of pure logicism—the logicist project is to show that one can

derive the laws of arithmetic *solely* from the laws of logic, not due to the arrangement of abstracta. So the idea that the laws of number are 'made true' by the objects Frege identifies as numbers is unacceptable for the logicist. Further still, Frege is a skeptic about truth-making itself, and thus would reject GROUND. In §1.3, I turn to my positive interpretation of Frege, particularly that he is an epistemological logicist, i.e. that he would have endorsed JUSTIFICATION or something relevantly similar. The primary insight is that Frege is, by and large, unconcerned with the truth of the laws of logic, because these laws are something which cannot be established (at least for the most part—Frege at least holds that not *all* laws of logic can be established without begging the question). Rather, the laws of logic are what constrain rational thought, and thus it is not possible to truly doubt them. Since we cannot truly doubt them, we are justified in an unconditional belief in them.

The upshot of my interpretation is that Frege's picture is more Kantian in nature than we might have initially assumed. Frege maps the limits of rationality, acknowledging that once those limits have been established it is impossible to question them without sacrificing said rationality. Then he proceeds to attempt to show that, given our rationally-required assumptions of logic, we can derive the laws of arithmetic. Thus, arithmetic cannot be rationally doubted, as it could if it were to rely on intuition or experience.

1.2 Metaphysical logicism

In some passages, Frege seems to be sympathetic to metaphysical logicism. In his critique of psychologism found in *Foundations of Arithmetic* and in *Basic Laws*, Frege criticizes the psychological logicians for their inappropriate metaphysical foundations of logic. A natural thought at this point would be that Frege intends to replace them with more appropriate metaphysical foundations. I take this to be a mistake.

The anti-psychologistic strain is found everywhere in Frege's thought. The majority of *Foundations of Arithmetic* is a discussion of the nature of mathematical facts, and argues

explicitly against the likes of Kant and Mill. It is there that Frege discusses numbers as mind-independent objects—a most anti-psychologisic position, and a metaphysical thesis on top of that. However, Frege's arguments in the second half of the introduction to *Basic Laws* are straightforwardly anti-psyschologistic.

Frege takes on explicit metaphysical commitments in the foreward to *Basic Laws*. While explaining the differences between his own view and the 'psychological logicians', he writes:

> I acknowledge a realm of the objective, non-actual, while the psychological logicians take the non-actual to be subjective without further ado. Yet it is utterly incomprehensible why something that has being independently of the judging subject has to be actual, i.e., has to be capable of acting, directly or indirectly, upon the sense ... Since the psychological logicians fail to appreciate the possibility of the objective non-actual, they take concepts to be ideas and thereby assign them to psychology. [9]

Here, Frege takes on a metaphysical commitment, particularly to numbers as objects as opposed to mere ideas in the head.[10]

A plausible reason for him to admit numbers into his ontology would be in order to provide referents for the signs which would appear in the *begriffsschrift.*[11] Frege is *not*, by admitting that numbers are objects, going to then claim that statements about number in arithmetic are thus *made true* by the way these numbers stand in relation to other numbers. This would be disastrous for Frege, on pain of inconsistency. Frege cannot hold that statements (or, more properly, *thoughts* in the Fregean sense) are made true by anything. Further, if Frege were to hold that the statements of arithmetic were made true, even in part, by numbers while also admitting that numbers are objects, Frege would have departed from logicism.[12] Frege's Platonism is independent of his logicism, and I posit

[10] An exegetical remark: by 'actual' I take Frege to earn something like 'physical' or 'natural' in the more contemporary philosophical senses of the terms—i.e. something is actual for Frege if we can have sensory experience of it, or if it is causally efficacious. Numbers plausibly are not, and thus for Frege are non-actual. Frege does not take 'actual' to be something like 'in the actual world.' Frege seems to have a dim view of possibility—c.f. his remarks in *Begriffsschrift.* According to Walter Hopp, Husserl makes a similar usage of 'actual' and 'real.'

[11] Frege makes it explicit that any signs which appear in the *begriffsschrift* ought to have referents, so that all the schemata refer to thoughts (as opposed to 'mock thoughts') and therefore have truth-values.

[12] This mistake is a common one. Quine, for instance, conflates Platonism with logicism in "On What There Is" [17]. But I do not think Frege conflated the positions.

that his ontology of numbers as objects played primarily a semantic role, giving meaning to number terms.

However, one may respond in a manner suggested by Linsky & Zalta [25]. In order to get the logicist project off the ground, one might have to expand what counts as logical. That is, some previously non-logical objects and relations might need to be considered logical. For example, the membership relation $\in$ and other set-theoretic notions have, at various points, been considered logical, though it is not clear that they pre-theoretically are.[13] Frege could admit certain objects into his ontology—numbers—and classify them as logical in character. This is not without precedent, since Frege appears to view value-ranges as logical (thus the recognition of Basic Law V). Or one could acknowledge that, if logicism *really* involves only the strictly logical (conceived in what we would now call 'classical', though this would be an anachronistic mistake in regards to Frege), then logicism is a non-starter, because mathematics involves, e.g., value-ranges, and such are not strictly logical.

At this point, I think it is best to hedge on the issue. What counts as logical is not, I think, particularly important in deciding whether or not Frege was a metaphysical logicist, when there is decisive evidence that Frege would have rejected GROUND. This evidence is found in Frege's remarks on truth.

1.2.1 Frege on truth and truth-making

The previous section shows that Frege had metaphysical commitments, but it did not establish that Frege was a metaphysical logicist. Crucially, Frege nowhere claims that statements about numbers are made true by the numbers themselves. I noted that this would be a departure from logicism. However, I recognize that this observation is contentious. I wish to provide a stronger argument against the metaphysical logicism interpretation.

Here is a quick and easy version of the argument. Frege nowhere provides an account of

[13] Hempel [13], for example, took $\in$ to be a logical relation, and thus thought of logicism as vindicated. It isn't, of course, clear that the membership relation is a logical one, nor is it clear that it is *not* strictly logical.

truth-making. Thus, he could not have endorsed GROUND and by extension metaphysical logicism. So, at the very least, Frege did not see himself as holding such a view or pursuing such a project. That is not to say, however, that such a project just did not occur to him. Frege gives several arguments against the notion of truth-making. I'll explore two of those arguments in this section. The first argument is to establish that no truth-making relation is sufficiently discriminatory. The second argument is to establish that truth-making is explanatorily null. Taken together, these arguments more than establish Frege's truth-making skepticism, and therefore he could not have endorsed GROUND and by extension metaphysical logicism.

1.2.1.1 Discrimination

In his landmark 1918 essay "Thoughts", Frege writes:

> It might be supposed ... that truth consists in a correspondence of a picture to what it depicts. Now a correspondence is a relation. But this goes against the use of the word 'true,' which is not a relative term and contains no indication of anything else to which something is to correspond ... A correspondence, moreover, can only be perfect if the corresponding things coincide and so just are not different things. [11]

The argument above is one against the idea of a truth-maker relation. The intended upshot is that there is no such relation—that is, no relation captures all the *desiderata* of a truth-maker theory. Frege seems to think that no relation will be sufficiently 'irrelative', we might say. Let's illustrate the passage with an example. Say we have some picture of a white house. The picture corresponds with many white houses— the residence of Barrack Obama, my childhood home, etc. If a picture is 'made true' in virtue of this correspondence, then it is made true by too many things.[14] Further, there is relative correspondence, or correspondence in degrees—but for Frege, there is no such thing as relative truth or degrees of truth.

[14]Set aside worries about pictures being truth-bearers—substitute 'representation' or 'thought' if need be.

The above argument, regardless of validity, demonstrates a skepticism on Frege's part of giving an analysis of the truth-making relation. We can understand the argument as a *reductio*: in order for a truth to be made true by some particular thing (state of affairs or whatever you prefer), the truth must stand in a particular relation to that thing. But correspondence is too weak a relation: a truth may correspond to many things, but only particular things are plausibly the right truth-makers. Correspondence is insufficiently discriminatory.

The upshot of this argument, as presented, is this. Call the relation which holds between all truths and their truth-makers the *truth-maker relation.* If the argument above is successful, then the worry is that for any candidate truth-maker relation `R`, there will be cases in which a state of affairs (or other candidate truth-maker) `s` and a truth-bearer `t` such that `Rts` holds, and yet `s` is not actually a truth-maker for `t`. Thus, `R` can't be a truth-maker relation. Call this the *Discrimination Problem.*

The Discrimination Problem, charitably read in Frege, is closer to a bet than a deduction. Frege wagers that there just aren't discriminatory enough relations to suffice for the truth-making relation (save for identity—and Frege does not seem to think that identity is a candidate truth-making relation). The Discrimination Problem is indeed a problem, or at least a worry. I take it as clear that Frege uses the argument to cast doubt on any analysis of truth-making. But it is not a *reductio* in any sense: we, as post-Fregeans, can look at the argument as making clear a *desideratum* for a theory of truth. The theory must be such that it (i) is weak enough that it explains the truth of all true truth-bearers in terms of their relations to their truth-makers and (ii) is strong enough that the truth-making relation does not hold between anything other than truths and their appropriate truth-makers.

But wait: doesn't Frege use truth-talk throughout his philosophy? How can this be so? I take it this kind of question comes from the thought that Frege's arguments imply skepticism about truth, not just about truth-makers. I take up this line of thought in §1.2.2.

1.2.1.2 Plenitude

The argument from "Thoughts" is similar to one found in "Logic," an unpublished essay dated to 1897.[15] There Frege writes:

> Now it would be futile to employ a definition in order to make it clearer what is to be understood by 'true'. If, for example, we wished to say 'an idea is true if it agrees with reality' nothing would have been achieved, since in order to apply this definition we should have to decide whether some idea or other did agree with reality. Thus we would have to presuppose the very thing that is being defined. The same would hold of any definition of the form 'A is true if and only if it has such-and-such properties or stands in such-and-such a relation to such-and-such a thing'. In each case in hand it would always come back to the question whether it is true that A has such-and-such properties, or stands in such-and-such a relation to such-and-such a thing. *Truth is obviously something so primitive and simple that it is not possible to reduce it to anything still simpler.* [10, emphasis mine]

One way to understand the argument is that there can never be a *sufficient* analysis of the relation: there must always be some further relation to explain why the first relation sufficed for truth. Another way is that the analysis entails an infinite number of relations, which is ontologicall intolerable (this is, I think, a more contemporary idea: Frege gives no indications that *he* saw the argument as being ontological in nature). These understandings of the argument are particularly strong because they allow for a natural generalization from *correspondence* theories of truth to *relational* theories of truth—an important difference from the argument in "Thoughts." Any analysis of the concept truth in which a thought is true iff it stands in a certain relation to a thing ('thing' being a neutral, general term) will be subject to the argument. Call this the *Plenitude Problem*.

It is more likely, I think, that Frege intended the argument as about explanatory power. Talk about presupposing truth in the analysis of truth is, I take it, strong evidence that Frege's worry is that any analysis of truth will have to rest on the assumption of some other truth—and thus the analysis failed to explain truth at all. In slogan form: any analysis of

[15] Though not the same. Recently, Heck [12] has made what I consider to be a mistake of conflating the two arguments. But as we see here, the argument in "Logic" does not give rise to the Discrimination Problem, but rather a problem not present in the argument from "Thoughts."

truth presupposes truth. Frege seemed to think that whenever one has made a judgment, one is already in the business of seeking truth. Truth is presupposed as the goal of inquiry (and, in a very real sense for Frege, it is the *object* of inquiry). To say that a judgment is true just is to make that judgment in the first place. Truth is so primoridal, it is the kind of thing that just cannot be analyzed, because analysis presupposes truth. Truth is special but not unique in this regard: the inference rule of *modus ponens* appears to occupy a similar space in Frege's thought.

1.2.2 The upshot of Discrimination and Plenitude

What are we to make of the above arguments? Frege is skeptical of the truth-making relation because of the Discrimination Problem, and he worries that the Plenitude Problem either multiplies relations without necessity, or that it renders the truth-making relation explanatorily null. But as I noted at the end of §1. 2.2.1, Frege has no problem engaging in truth-talk, and in fact truth may be a central notion in his philosophy of logic.[16] Frege is not a truth *eliminativist*. The urge to say that Frege makes no use of truth, or at least ought not to, comes from the acceptance of a false dilemma. The dilemma: either truth can be analyzed, or there is no such thing as truth. Frege belives that no substantive analysis (or, to use only slightly anachronistic jargon, 'real definition') of truth can be given, yes. But he does not believe that from this observation truth is rendered unnecessary or particularly problematic. Frege is a truth *primitivist*. He believes that there is nothing that 'explains' the truth of a thought—though all thoughts do denote the True, where the True is a special kind of object.

> One might be tempted to regard the relation of the thought to the True not as that of sense to *Bedeutung*, but rather as that of subject to predicate. One can, indeed, say: 'The thought that 5 is a prime number is true'. But closer examination shows that nothing more has been said than in the simple sentence '5 is a prime number'. The truth claim arises in each case from the form of the assertoric sentence, and when the latter lacks its usual force, e.g., in the mouth

[16] C.f. Heck, "Frege and Semantics" [12] and Stanley, "Truth and Metatheory in Frege" [21]. For an opposing view, see Weiner, "Semantic Descent" [23].

> of an actor upon the stage, even the sentence 'The thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number.' [8]

The argument here seems to be that truth is not a property that can be predicated of thoughts because 'is true' is semantically null—here Frege is reminiscent of the deflationists.[17] All true thoughts have the True as their *Bedeutung*.[18] Frege's belief that all true thoughts denote the True seems to arise from his belief in compositionality (an idea for which he is to be commended). But this 'denotational analysis' of true thoughts is not an analysis of their truth in any real sense—that a thought denotes the true does not really explain its truth-value.[19] Consider an analogy with Moore's Open Question Argument [16]. Moore's argument is not that since good resists a real definition, there is no such thing. This is because Moore recognizes the falsity of the analysis dilemma. Instead, good is indefinable—but it is no less a part of world because of this. Truth holds a similar status in Frege's thought.[20][21]

1.2.3 From primitivism to the denial of metaphysical logicism

Metaphysical logicism and primitivism about truth make for strange bedfellows. If one wishes to say that some truths are grounded in some class of facts, one needs to provide

[17]Paul Horwich [15] claims his deflationary view of truth is akin to Frege's. This isn't right, because Frege just isn't a deflationist. For Horwich, the deflationist thinks that truth can play no special theoretical role because it is lacking a substantive 'nature.' But Frege puts truth to work in his philosophy—and thus he isn't a deflationist.

[18]Thoughts—in the sense of something like a proposition—are, for Frege, the truth-bearers.

[19]It is clear Frege saw it this way, as he writes in "Logic": " Truth is obviously something so primitive and simple that it is not possible to reduce it to anything still simpler." If Frege believed truth could be explained in terms of denotation, then he would have said no such thing. Obviously then, denoting the True is a property all true thoughts have, but it does not suffice for truth. Admittedly, this makes Frege's use of 'refers to the True' as a truth predicate strange—but I am not interested, in this current piece, on Frege's semantics of truth-value ascriptions.

[20]Joan Weiner [24] has recently argued that Frege intends his views about truth to only be applicable to sentences in Frege's formalized language. Such an interpretation seems to overlook the care with which Frege considers natural language in "On *Sinn* and *Bedeutung*." It would hardly be worthwhile for Frege to consider, e.g., the problem of sentences under the scope of epistemic operators if he thought his analysis was about truth-for-*begriffsschrift*, not truth *simpliciter*.

[21]Contrasting Frege with Moore is helpful in making clear Frege's view of truth. Frege denies that any meaningful analysis of truth can be given because truth is primitive. Moore provides an analysis of truth: truth is correspondence to the facts, but the correspondence relation is primitive.

reasons. A natural reason: a certain relation R holds between these truths and members of the class of facts. But if one is a primitivist about truth, then one can't say this—because truth can't be explained. If one does wish to say this, then one has either abandoned primitivism or changed the subject. I doubt Frege is guilty of either.

1.2.4 A problem from *Foundations*

Frege, in §2 of *Foundations of Mathematics* writes:

> The aim of proof is, in fact, not merely to place the truth of a proposition beyond all doubt, but also to afford us insight into *the dependence of truths upon one another.* After we have convinced ourselves that a boulder is immovable . . . there remains the further question, what is it that supports it so securely? [7, emphasis mine]

This is the most difficult passage in Frege's corpus for my interpretation. Frege has clearly made a claim eerily similar to GROUND—and thus he may be a metaphysical logicist.

However, the above passage was written before 1884. The arguments from "Thoughts" and "Logic" come 13 and 34 years later, respectively. Frege's mature thought on truth and truth-making simply had not come to be by 1884. While this is perhaps an unexciting interpretive move, I think it is the right one. *Foundations* is particularly insightful in its critiques of psychologism and empiricism—Frege clears the way for the development of his own philosophy of mathematics, which he then develops over the rest of his life. The arguments for the indefinability of truth and the incoherence of the truth-making relation come afterward, and with these arguments Frege turns to his epistemological project.

Richard Heck [12] makes a similar point. It is common in Frege scholarship to pull various passages from across Frege's corpus in an attempt to provide a cohesive picture of his project. While the project, in a minimal sense, remains constant (Frege remained committed to logicism), the philosophical import of logicism was not immediately clear to him. Frege's metaphysical ambitions are found in *Foundations*, a relatively early work, but these ambitions do not appear in *Basic Laws* or the later articles. With such a difference noted, the hypothesis that Frege made progress after 1884 becomes even more plausible.

1.3 Epistemological logicism

We can now turn to our second question: would Frege have endorsed JUSTIFICATION, or at least something reasonably similar? Throughout, I'll argue for the affirmative answer.

1.3.1 The attack on psychologism, redux

Just as we turned to the critique of psychologism to uncover some of Frege's metaphysical commitments, we can turn to it to see Frege's characterization of his project as epistemological, particularly in *Basic Laws*. That is, psychologism is not just metaphysically intolerable for conflating mental states with the objects of those mental states—psychologism robs mathematics of its certainty as well. Frege, after attacking psychologism, wants to provide certainty for mathematics. He does this not by giving a new metaphysical basis, but by means of proof. Proofs provide certainty so long as one is certain of the starting assumptions and the rules of inference—and it is here Frege is exceptionally clear, thus the need for his *begriffsschrift* in the first place.

Frege writes in the second half of the forward to *Basic Laws* that the "obstacle to the influence of [his] book" is not the strange symbolism which it employs (he defends his formalism at length earlier in the forward), but rather the "ruinous incursion of psychology into logic" [9]. It is this incursion that leads some to think that, for example, there is no difference between representation and thing represented—relatedly, there is then no distinction between laws of thought in the normative and descriptive senses. Psychology describes inferences made; it cannot say how the inferences ought to be made. Frege is rightly opposed to this idea, as when he writes that "being true is different from being taken to be true, be it by one, be it by many, be it by all, and is in no way reducible to it" [9]. The laws of thought, as characterized in *Basic Laws*, are normative—they govern thought (or better yet *thinking*). That is, the proceeding from one idea to another is appropriate or correct insofar as it complies with the laws of thought. This is a distinctly epistemological notion: it deals in justification, licensed inference, and rationality. Thus when Frege writes

that about the possibility (or, rather, impossibility) of rejecting certain logical laws like the law of identity (which Frege takes to be "Every object is identical to itself"), he says:

> [T]his impossibility, to which we are subject, of rejecting the law does not prevent us from supposing beings who do so; but it does prevent us from supposing that such beings do so rightly; and it prevents us, moreover, from doubting whether it is we or they who are right. [9]

Because of the kinds of creatures we are—bound by the laws of logic—we cannot imagine disbelieving the law of identity, though we can imagine disbelieving creatures, so-called 'logical aliens'. Importantly, though, we cannot imagine *correct* logical aliens. It is beyond what we are capable of thinking. This does not mean that the laws of logic are 'made true' by this inability—this would be to put the cart before the philosophical horse. Instead, for Frege it points to a constraint on rational thought. His project, then, is one which accepts these constraints and operates within them. Taking the constraints on rational thought as primitive, can he prove the laws of arithmetic? Frege takes the answer to be a resounding *yes*, and it is to the specifics of the project that he dedicates *Basic Laws*.

1.3.2 The project in *Basic Laws*

Frege often begins his works by pointing to the insufficiency of similar projects. When framing his *Begriffschfrift*, e.g., Frege seeks to show the clarity of his concept-script by showing the problematic aspects of Boole's calculus.[22] *Foundations of Arithmetic* is primarily a criticism of his Frege's predecessors. In *Basic Laws*, Frege characterizes his project as in the same vein as Dedekind's, though different in degree of success. Why is Frege's project more successful? He writes:

> My purpose demands some divergences from what is common in mathematics. Rigour of proof requires, as an inescapable consequence, an increase in length. Whoever fails to keep an eye on this will indeed be surprised how cumbersome our proofs often are of propositions into which he would suppose he had an immediate insight, through a single act of cognition. This will be especially

[22] C.f. [5] and [6].

> striking if one compares Mr. Dedekind's essay, *Was sind und was sollen die Zahlen?*, the most thorough study I have seen in recent times concerning the foundations of arithmetic. It pursues, in much less space, the laws of arithmetic to a much higher level than here. This concision is achieved, of course, only because much is not in fact proven at all . . . Mr. Dedekind too is of the opinion that the theory of numbers is a part of logic; but his essay barely contributes to the confirmation of this opinion since his use of the expressions "system", "a thing belongs to a thing" are neither customary in logic nor reducible to something acknowledged as logical.[23] [9]

The above passage is telling for several reasons. Dedekind makes uses of expressions which are not common in mathematics or in logic—and so, it seems, Dedekind cannot be said to be advancing the logicist project. This contrasts with Frege, who often points to mathematical practice as evidence for accepting some introduced notion—he claims, for example, that Basic Law V (the law of value-ranges) is implicit in mathematical practice, and he uses this to assuage any doubts about its status as a logical law.[24] So the logicist project, for Frege, involves taking already accepted notions in logic and mathematics and putting them to use, *not* the introduction of new notions.[25][26]

Dedekind's project is also insufficient because there is *room for doubt*, not because of any metaphysical concerns. Dedekind, Frege says just a few lines down, misses intermediate steps in chains of inferences and "merely gestures at them." That is, in order for the proofs to be beyond doubt they must be 'gapless':

> The gaplessness of the chains of inferences contrives to bring to light each axiom, each presupposition, hypothesis, or whatever one may want to call that on which a proof rests; and thus we gain a basis for an assessment of the *epistemological nature of the proven law.* [9, emphasis mine]

[23] Frege does speak about a reduction at the end of the quoted passage, and reduction is a decidedly metaphysical notion. But here I think by 'reducible' Frege is referencing his idea of definition and stipulation found in *Begriffsschrift.* We can define new notions or ideas in terms of simpler ones—Dedekind's error is introducing non-logical, non-mathematical notions into his alleged proofs.

[24] Obviously Frege made a misstep here. However, at this point in my interpretation, I am interested in what Frege took himself to be doing, not what he succeeded in doing.

[25] Truth-making, it seems to me, would be one such new notion.

[26] I can imagine a critic saying in response to this: Frege *does* introduce new notions, such as *Sinn* and *Bedeutung*, the True and the False as objects, and thoughts. But the introduction of these notions is either just the making explicit of something already implicit, or a theoretically-driven introduction that other plausible concerns motivate.

By giving gapless proofs, we 'gain a basis for an assessment of the *epistemological nature* of the proven law.' That is to say, the proof tells us nothing about the source of its truth, or its grounding, or its truth-maker—it tells us that the proven law is *beyond doubt*. It does not tell us that the law is true. Psychologism does neither—it cannot assuage such worries and doubts, because (as Frege argues) it appears to collapse into subjective idealism. Frege's procedure can. The goal of the project is to rid us of any doubt about the proven laws.

Patricia Blanchette [1] raises a worry for Frege's project at this point. The problem for Frege's project is one of conceptual analysis—for each step of analysis, Frege faces a chance of doubt in the analysis. It is precisely this problem Frege must overcome in order for the problem to be successful. The 'gaplessness' of the chains of inference does not protect the project from such a worry. Frege's project is one that builds upon former definitions in service of new ones—that is, Frege's definitions become more complex when written without his definitional shorthand. With each new 'layer' of definition, Frege must perform conceptual analysis sufficient to convince the reader that the definition is a good one.

1.4 An advantage of epistemological logicism

We've seen that there are good reasons to think that Frege was an epistemological logicist, i.e. that he would have endorsed JUSTIFICATION. This is an interpretive point. We can step back a bit and now ask: what other, non-obvious advantages are their to pursuing epistemological logicism as opposed to metaphysical logicism on Fregean grounds?

The basic upshots of the projects are of course the same; this is captured by Frege's adage that 'the theory of numbers is a part of logic.' But the more philosophical upshots differ depending on one's conception of logicism (as we would expect). Particularly, epistemological logicism appears to solve a problem that metaphysical logicist cannot given some Fregean assumptions.

By putting the laws of logic beyond doubt, Frege makes an interesting move. He seems

to put the truth of the laws of logic to the side—it isn't for us to determine if the laws of logic are *true*, since we cannot think otherwise. This is evidence enough that we can assume them with no concern.

Frege actually solves a problem here, a problem that it is not clear he was aware of. The problem is that there may be qualitatively indiscernible mental states that, if Frege were to be concerned about the truth of the laws of logic, which would act as evidence to doubt the laws of logic. But this is precisely the kind of result we, following Frege, would like to avoid. I explain the problem in more detail below.

First, let's stipulate some terminology. Let's say that we know* some thought or proposition ϕ iff one believes ϕ, is unable to doubt ϕ, and ϕ is true. Knowing* is a subset of knowing *simpliciter*, because in order to know ϕ one must believe ϕ and ϕ must be true—whenever one knows*, one knows, but the converse does not hold. If our relationship to the laws of logic had anything to do with truth, we would stand in the knowing* relation to them. Consider further a different mental state, schmowing. One schmows a proposition or thought ϕ iff one believes ϕ, one is unable to doubt ϕ, and ϕ is in fact false. Assuming that ϕ cannot be both true and false, then it follows that one never knows* ϕ while also schmowing ϕ.

Set aside worries about whether or not there is an actual distinct *kind* of mental state as knowing*, or an actual distinct *kind* of mental state as schmowing. Surely there are mental states that satisfy the descriptions.[27] All we need is that there may be mental states relevantly similar to knowing* and schmowing. Now, one can never tell if one is knowing* or schmowing—that is, one can know one is either knowing* or schmowing, but one cannot be sure which of the two states one is in. There will be cases where one will claim that one is either knowing* or stemming (whenever one cannot rationally doubt some ϕ), but if one is aware of the possibility of schmowing, then one cannot justifiably assert that one knows* as opposed to schmows.

[27] There is a paradox lurking here: I am claiming that there are certain beliefs I have which are impossible to doubt, and yet claiming that some of them are false. But I think the issue can be discussed without giving too much thought to such a paradox, as it is not relevant to the current discussion.

The existence of schmowing should serve as a defeater for any belief in which one cannot rationally doubt. There is, after all, *no way* of knowing whether one knows* or schmows. So, if one is primarily interested in the truths of the laws of logic (as someone who endorses GROUND would be), then one is faced with the possibility of only schmowing the laws of logic, and thus they cannot be beyond doubt.

The epistemological logicist has no such problem. After all, truth isn't the relevant question about the laws of logic. Recall that the epistemological logicist starts by asking the question: in virtue of what am I justified in being fully confident in the laws of arithmetic specifically and (perhaps) mathematics more generally? The answer is that they follow from the laws of logic, and we cannot rationally doubt the laws of logic—they constrain rational thought. Thus, we cannot doubt the laws of arithmetic. If it turns out that the laws of logic are false or fail to be fully general, this is irrelevant. No matter what, whenever we consider a particular law of logic, we cannot regard it as false. Thus, I take it, the epistemological logicist does not solve the knowing*/schmowing problem—he simply makes it non-applicable to the particular domain.

1.5 Conclusion

Let's review. I have proposed a new distinction between types of logicism: metaphysical and epistemological logicism. I've characterized the views by their acceptance of GROUND and JUSTIFICATION respectively. I then argued that Frege would reject GROUND and accept JUSTIFICATION. There are two major goals of inquiry: truth and justification. Frege does not attempt to show why the laws of arithmetic are true. He is showing why we are justified in believing that they are true. Now, obviously justification is 'tied up' with truth in an important way, as belief is about truth. But we can remain neutral as to the grounds of a truth while seeking justification for the belief. And it is justification for beliefs in and about mathematics that Frege is seeking. Interpreting Frege in this way makes sense of his view of truth, showing how it connects to the larger project (particularly,

by endorsing primitivism, Frege closes off the option of endorsing GROUND). And viewing Frege's project as primarily epistemological in character solves the knowing*/schmowing problem raised in §1.4.

The taxonomy provided is a useful one. There are two major goals of inquiry: truth and justification. Frege does not attempt to show why the laws of arithmetic are true. He is showing why we are justified in believing them. Now, obviously justification is 'tied up' with truth in an important way, as belief is about truth. But we can remain neutral as to the grounds of a truth while seeking justification for the belief. And it is justification for beliefs in and about mathematics that Frege is seeking. There is a narrative, due mostly to Michael Dummett (e.g. [3] and [4]), that Frege was a philosopher of language, that his central insight was to look for the objects of thought outside of the mind—and I think this narrative isn't without its virtues. But thinking of Frege in this way overlooks Frege's central project: to put mathematics beyond rational doubt, to show that it is *a priori*, and (in service of this project) to show that it follows from the laws of logic.

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